APPLICATION OF MCT FAILURE CRITERION USING EFM

1 Summary

This document describes the work done in NUS to implement the Multi-Continuum Theory (MCT) of composite failure for damage progression analysis using the Element-Failure Method (EFM) in ABAQUS. The EFM-MCT has been implemented in ABAQUS using a user defined element subroutine EFM. Comparisons have been made between the analysis results using EFM-MCT code and HELIUS:MCT™ code which uses material property degradation method (MPDM). It is found that the conventional EFM-MCT code is less efficient than the HELIUS:MCT™ code. This is due to the iterative nature of the EFM code in finding the solution. A non-iterative element-failure method (N-EFM) is then developed to improve the computational efficiency. This N-EFM-MCT has been implemented in an in-house code and comparison of computational efficiency has been made between N-EFM and MPDM.

2 Multi-continuum theory

Multi-continuum theory (MCT) [1-15] is a micromechanics-based failure theory applicable to fiber reinforced plastic which is based on phase averaging of stress and strain. Based on the averaging of stress and strain of constituent matrix and fiber in a composite, a relationship between stress and strain of the composite and the stress and strain of the constituents can be established, i.e. constituent (matrix and fiber) stress and strain can be calculated from the composite stress and strain. The failure can then be evaluated based on the constituent stress.

The phase averaging of stress and strain in MCT for a limited representative volume is expressed as [1]:

$$\boldsymbol{\sigma}^c = \varphi_f \boldsymbol{\sigma}^f + \varphi_m \boldsymbol{\sigma}^m \tag{1}$$

$$\boldsymbol{\epsilon}^c = \varphi_f \boldsymbol{\epsilon}^f + \varphi_m \boldsymbol{\epsilon}^m \tag{2}$$

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14. ABSTRACT

This document describes the work done in National University of Singapore to implement the Multi-Continuum Theory (MCT) of composite failure for damage progression analysis using the Element-Failure Method (EFM) in ABAQUS. The EFM-MCT has been implemented in ABAQUS using a user defined element subroutine EFM. Comparisons have been made between the analysis results using EFM-MCT code and HELIUS:MCT? code which uses material property degradation method (MPDM). It is found that the conventional EFM-MCT code is less efficient than the HELIUS:MCT? code. This is due to the iterative nature of the EFM code in finding the solution. A non-iterative element-failure method (N-EFM) is then developed to improve the computational efficiency. This N-EFM-MCT has been implemented in an in-house code and comparison of computational efficiency has been made between N-EFM and MPDM.

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where $\boldsymbol{\sigma}^c$,$\boldsymbol{\sigma}^f$, and $\boldsymbol{\sigma}^m$are 6 x 1 vectors that represent respectively composite , fiber and matrix average stress; $\boldsymbol{\epsilon}^c$,$\boldsymbol{\epsilon}^f$, and $\boldsymbol{\epsilon}^m$ are 6 x 1 vectore that represents composite , fiber and matrix average stress matrices, respectively; while $\boldsymbol{\varphi}_f$ and $\boldsymbol{\varphi}_m$ are respectively fiber and matrix volume fraction.

MCT uses the following linear elastic constitutive relationship between stress and strain:

$$\boldsymbol{\sigma}^c = \boldsymbol{C}_c(\boldsymbol{\epsilon}^c - \boldsymbol{\epsilon}_0^c) = \boldsymbol{C}_c(\boldsymbol{\epsilon}^c - \theta\boldsymbol{\alpha}_c) \tag{3}$$

$$\boldsymbol{\sigma}^f = \boldsymbol{C}_f\left(\boldsymbol{\epsilon}^f - \boldsymbol{\epsilon}_0^f\right) = \boldsymbol{C}_f\left(\boldsymbol{\epsilon}^f - \theta\boldsymbol{\alpha}_f\right) \tag{4}$$

$$\boldsymbol{\sigma}^m = \boldsymbol{C}_m(\boldsymbol{\epsilon}^m - \boldsymbol{\epsilon}_0^m) = \boldsymbol{C}_m(\boldsymbol{\epsilon}^m - \theta\boldsymbol{\alpha}_m) \tag{5}$$

where $\boldsymbol{C}_c$, $\boldsymbol{C}_f$, and $\boldsymbol{C}_m$ represent 6 x 6 stiffness matrices of composite, fiber and matrix, respectively; θ is temperature change; $\boldsymbol{\alpha}_c$, $\boldsymbol{\alpha}_f$, $\boldsymbol{\alpha}_m$ are 6 x 1 vectors of composite, fiber and matrix thermal expansion coefficients, respectively. By using equations (1) to (5), relationships between composite stress and strains with constituent (fiber and matrix) stress and strains can be determined; hence constituent stress and strain can be calculated from composite stress and strain.

Once the constituent stress and strain are determined, MCT evaluates composite failure based on its constituent stress. The failure criterion used in MCT for matrix can be expressed as [1]:

$$\pm A_1^m (I_1^m)^2 - \pm A_2^m (I_2^m)^2 + A_3^m I_3^m + A_4^m I_4^m - \pm\pm A_5^m I_1^m I_2^m = 1 \tag{6}$$

while the failure criterion for fiber is

$$\pm A_1^f \left(I_1^f\right)^2 + A_4^f I_4^f = 1 \tag{7}$$

where

$$I_1^m = \sigma_{11}^m \tag{8}$$

$$I_2^m = \sigma_{22}^m + \sigma_{33}^m \tag{9}$$

$$I_3^m = (\sigma_{22}^m)^2 + (\sigma_{33}^m)^2 + 2(\sigma_{23}^m)^2 \tag{10}$$

$$I_4^m = (\sigma_{12}^m)^2 + (\sigma_{13}^m)^2 \tag{11}$$

$$I_1^f = \sigma_{11}^f \tag{12}$$

$$I_4^f = \left(\sigma_{12}^f\right)^2 + \left(\sigma_{13}^f\right)^2 \tag{8}$$

${}^{\pm}A_1^m$, ${}^{\pm}A_2^m$, A_3^m, A_4^m, ${}^{\pm\pm}A_5^m$, ${}^{\pm}A_1^f$, and A_4^f are failure coefficients that can be derived from tensile, compression, and shear test data of the composite [1].

3 Element failure method

This section describes the conventional iterative element-failure method (EFM) and the modified non-iterative element-failure method (N-EFM).

3.1 Iterative Element-Failure Method

Element-failure method (EFM) is a damage progression algorithm in which the effect of damage in mechanical behavior is affected through nodal forces in finite element (FE) analysis. The concept was first conceived for dynamic fracture in metals [16], but the modified EFM was found useful for the analysis of impact damage in fiber-reinforced composites [17], damage progression in quasi-statically loaded three point bend composite laminates [18] and ultimate strengths of open-hole tension composite laminates [19]. The manner by which these effects due to damage translate to the effective nodal forces will in general depend upon the damage evolution law appropriate to the local mode of damage experienced by the composite material, as well as the FE formulation. Traditionally, when damage is assumed to have occurred within an element of a material, it is reasonable to assume that the stiffness matrix of the element is altered to reflect the damaged state. As the following derivation shows, there are explicit relations between the nodal forces and the elastic stiffness of an FE.

The force-stiffness relation for an FE is given by

$$\boldsymbol{K}\,\boldsymbol{u} = \boldsymbol{f} \tag{9}$$

where $\boldsymbol{u}$ is the vector of nodal displacements, $\boldsymbol{f}$ the vector of nodal forces and the element stiffness matrix of undamaged material. In FE analysis, the stiffness matrix $\boldsymbol{K}$ is calculated by integrating over the element domain:

$$\boldsymbol{K} = \int_{\Omega} \boldsymbol{B}^T \boldsymbol{C}\, \boldsymbol{B}\ \partial\Omega \tag{10}$$

where $\boldsymbol{B}$ is the derivative of element shape function and $\boldsymbol{C}$ is the material stiffness matrix.

In conventional progressive damage algorithm which uses material property degradation method (MPDM), element or material failure is modeled by modifying the material stiffness $\boldsymbol{C}$ and hence the element stiffness matrix $\boldsymbol{K}$. EFM, however, models the failure by directly modifying the force vector $\boldsymbol{f}$. Consider an FE of an undamaged composite material (Figure 1 (a)), experiencing a set of nodal forces, which have been obtained from the FE solution of the problem. On the other hand, in an FE containing damaged material, the load-bearing capacity of the FE will be compromised, very likely in a directionally and spatially dependent manner. If much of the damage consists of transverse matrix microcracks, it is reasonable to assume that the FE will have reduced load-bearing capacity in the direction transverse to the fibers (Figure 1 (b)). In conventional MPDM, this reduction is achieved by reducing or zeroing certain pertinent material stiffness properties of the damaged FE. In the EFM, however, the reduction is effected by applying a set of external nodal forces such that the net internal nodal forces of elements adjacent to the damaged element are reduced or zeroed (the latter if complete failure or fracture is implied (Figure 1 (c)). The decision whether to fail an element is guided by a suitable failure theory. The required set of applied nodal forces to achieve the reduction within each step is determined by successive iterations until the net internal nodal forces (residuals) of the adjacent elements converge to the desired values. Note that it is not the internal nodal forces of the damaged element that is zeroed (for the case of complete failure (Figure 1 (c)), but the net internal nodal forces of adjacent elements. Thus the ‘stresses’ within the failed element no longer have physical meaning, although compatibility may be preserved. This process leaves the original (undamaged) material stiffness properties unchanged, and is thus computationally efficient as every step and iteration are simply analysis with the updated set of applied nodal forces. For this reason, it may also be called the nodal force modification method. Hence, no reformulation of the FE stiffness matrix is necessary.

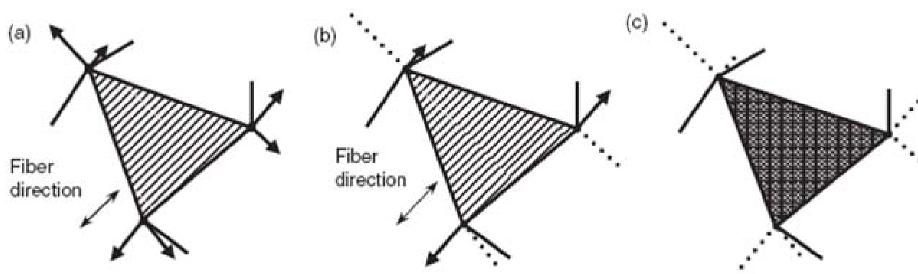


Figure 1 (a) FE of undamaged composite with internal nodal forces ; (b) FE of composite with transverse matrix cracks. Components of internal nodal forces transverse to fiber direction are modified; (c) Completely failed or fractured element. All net internal nodal forces of adjacent elements are zeroed

3.2 Non-Iterative Element-Failure Method (N-EFM)

Non-iterative element failure method (N-EFM) is focused on implementation of damaged elements with direct solution and implicit external nodal forces applied to the damaged elements. Compared with the traditional FEM with MPDM, N-EFM doesn't require frequent assembly and decomposition of global stiffness matrix in each load increment. The basic equation in N-EFM is derived from the general FEM as:

$$\tilde{\boldsymbol{K}}\boldsymbol{a}_I = \tilde{\boldsymbol{F}}_I \tag{16a}$$

where

$$\begin{cases} \tilde{\boldsymbol{K}} = \boldsymbol{I} - \boldsymbol{K}_0^{-1}\Delta\boldsymbol{K} \\ \tilde{\boldsymbol{F}}_I = \boldsymbol{K}_0^{-1}\boldsymbol{P}_I \end{cases} \tag{17}$$

and $\boldsymbol{K}_0$ is the initial stiffness matrix without damage, which keeps constant through the analysis; $\Delta\boldsymbol{K}$ is the stiffness matrix for damaged elements, which is updated with failure progression; $\boldsymbol{a}_I$ and $\boldsymbol{P}_I$ are the displacements and known loads in the *I*th increment.

In Eq.(17), only one-time LU decomposition (e.g. Cholesky factorization, $\boldsymbol{K}_0 = \boldsymbol{L}\boldsymbol{L}^{\mathrm{T}}$) at the beginning of the analysis is required and then stored for use in subsequent computations. $\tilde{\boldsymbol{F}}_I$

can be easily obtained by forward and backward substitutions with the LU decomposition. Additionally, $\boldsymbol{K}_0^{-1}$ is also not explicitly calculated and only columns related to damaged elements are required, which are obtained by a same method with that of obtaining $\tilde{\boldsymbol{F}}_I$. The multiplication $\boldsymbol{K}_0^{-1}\Delta\boldsymbol{K}$ in Eq.(17) accordingly takes the form as:

$$\boldsymbol{K}_0^{-1}\Delta\boldsymbol{K} = \begin{bmatrix} 0 & \cdots & k_{1j} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & k_{ij} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & k_{nj} & \cdots & 0 \end{bmatrix}_{(n\times n)} \tag{18}$$

where only columns related to damaged elements are non-zero.

As $\boldsymbol{K}_0$ is constant through the analysis, only columns in $\boldsymbol{K}_0^{-1}$ related to new damaged elements are calculated in each load increment and then stored for use in subsequent load increments. After $\tilde{\boldsymbol{K}}$ and $\tilde{\boldsymbol{F}}_I$ are obtained, Eq.(16a) can be rearranged with the following formulation:

$$\begin{bmatrix} \tilde{\boldsymbol{K}}_d & \boldsymbol{0} \\ \tilde{\boldsymbol{K}}_u & \boldsymbol{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_d \\ \boldsymbol{a}_u \end{bmatrix}_I = \begin{bmatrix} \tilde{\boldsymbol{F}}_d \\ \tilde{\boldsymbol{F}}_u \end{bmatrix}_I \tag{16b}$$

where $\tilde{\boldsymbol{K}}_d$ and $\tilde{\boldsymbol{K}}_u$ are condensed matrices related to damaged and undamaged elements, respectively, and the subscript "*d*" and "*u*" denote damaged elements and undamaged elements, respectively.

Then the large-size problem described by Eq.(16a) or Eq.(16b) can be reduced to a small-size problem described by the following equation:

$$\tilde{\boldsymbol{K}}_d \boldsymbol{a}_d = \tilde{\boldsymbol{F}}_d \tag{19}$$

where the solution is only focused on damaged elements and the problem size is dependent on the number of damaged elements.

After the displacement related to damaged elements, $\boldsymbol{a}_d$, is obtained, the displacement related to undamaged elements, $\boldsymbol{a}_u$, can be obtained straightforward with substituting $\boldsymbol{a}_d$ into the following equation according to Eq.(16b):

$$\boldsymbol{a}_u = \tilde{\boldsymbol{F}}_u - \tilde{\boldsymbol{K}}_u \boldsymbol{a}_d \tag{20}$$

In N-EFM, the global stiffness matrix for damaged elements, $\Delta\boldsymbol{K}$, as indicated in Eq.(15), is assembled with the element stiffness matrix for damaged elements, $\Delta\boldsymbol{K}^e$, which is determined by the following equations with MCT:

$$\Delta\boldsymbol{K}^e = \begin{cases} \boldsymbol{K}^e, \text{ if fibre fails} \\ \boldsymbol{K}_m^e, \text{ if matrix fails} \end{cases} \tag{21}$$

$$\begin{cases} \boldsymbol{K}^e = \sum \boldsymbol{B}^{\mathrm{T}} \bar{\boldsymbol{D}} \boldsymbol{B} \\ \boldsymbol{K}_m^e = \sum \boldsymbol{B}^{\mathrm{T}} \bar{\boldsymbol{D}}_m \boldsymbol{B} \end{cases} \tag{22}$$

where $\bar{\boldsymbol{D}}$ and $\bar{\boldsymbol{D}}_m$ are full and reduced global material stiffness matrices, respectively; $\bar{\boldsymbol{D}}$ and $\bar{\boldsymbol{D}}_m$ are obtained with transformation of full and reduced local material stiffness matrices, $\boldsymbol{D}$ and $\boldsymbol{D}_m$, respectively, and

$$\left(D_m\right)_{ij} = \begin{cases} 0, & \text{if } i, j = 1 \\ D_{ij}, & \text{if } i, j \neq 1 \end{cases} \tag{23}$$

In order to speed up the computation of N-EFM, the initial stiffness matrix without damage in Eq.(17), $\boldsymbol{K}_0$, can be replaced by a reassembled stiffness matrix after every J_0 increments, $\boldsymbol{K}_{J_0}$, and $\Delta\boldsymbol{K}$ is also replaced by that after every J_0 increments, $\Delta\boldsymbol{K}_{J_0}$, which is assembled only for new damaged elements in every J_0 increments. Then the N-EFM equation can be modified as:

$$\tilde{\boldsymbol{K}}_{J_0} \boldsymbol{a}_I = \left(\tilde{\boldsymbol{F}}_{J_0}\right)_I \tag{24}$$

and the reassembled stiffness matrix, $\boldsymbol{K}_{J_0}$, is determined by the following equation:

$$\boldsymbol{K}_{J_0} = \sum^{m_t - m_d} \boldsymbol{K}^e + \sum^{m_d} \boldsymbol{K}_d^e \tag{25}$$

where m_t and m_d are total number of elements and number of damaged elements, respectively, and

$$\boldsymbol{K}_d^e = \begin{cases} \boldsymbol{K}'^e, & \text{if fibre fails} \\ \boldsymbol{K}_m'^e, & \text{if matrix fails} \end{cases} \tag{26}$$

and the element stiffness matrix $\boldsymbol{K}'^e$ and $\boldsymbol{K}_m'^e$ are determined by the local material stiffness matrices D'_{ij} and $\left(D'_m\right)_{ij}$ with appropriate transformations, which are defined as:

$$D'_{ij} = d_M D_{ij} \approx 0 \tag{27}$$

$$\left(D'_m\right)_{ij} = \begin{cases} D_{ij}, & \text{if } i, j = 1 \\ d_M D_{ij}, & \text{if } i, j \neq 1 \end{cases} \tag{28}$$

where $d_M \approx 0$ is a material degradation factor.

4 Implementation of EFM-MCT in ABAQUS

EFM-MCT has been implemented in ABAQUS using user defined element subroutine UEL. The exactly same MCT failure criterion used in the HELIUS:MCT™ code [1] is used in the EFM-MCT code. In this code, failure of each element is examined in its centroid. EFM nodal force modification is performed whenever any element is considered failed. If matrix in an element is considered failed, the nodal force is modified so that nodal force due to E_2, E_3, E_{12}, E_{13}, and E_{23} are negated. If fiber in an element is considered failed, the nodal force is modify such that it negates the stiffness of the element.

Damage analysis using MCT requires the calculation of matrix, fiber, and composite coefficients which include stiffness, Poisson's ratio, and failure coefficients before the analysis begins. HELIUS:MCT™ uses Helius material manager to calculate these coefficients based on the measured composite stiffness, Poisson's ratio and strength, and the measured constituent (matrix and fiber) stiffness and Poisson's ratio. The EFM-MCT code that is developed in this study also uses Helius material manager to calculate the coefficients.

Damage analysis using EFM-MCT requires two simulations: the first simulation is the actual progressive damage analysis, while the second simulation is dummy analysis for visualization purpose. The second simulation is needed because ABAQUS does not support visualization for elements from UEL subroutine. Thus, in this code, element failure information from the 1st analysis is stored in a text file. This text file is then read in the second analysis and the element failure information is stored in ABAQUS native elements so that it can be visualized in ABAQUS.

5 Comparison between HELIUS:MCT™ and EFM-MCT

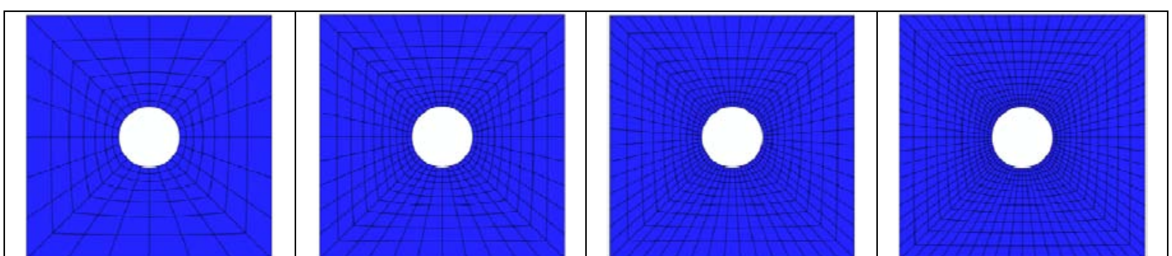
Open hole tension (OHT) models are analyzed using HELIUS:MCT™ and EFM-MCT codes in ABAQUS in order to compare the two methods. The plate is 76.2 mm x 76.2 mm in size and it has a 19.05 diameter central hole. Carbon/epoxy IM7/5250-4 composite laminate with stacking sequence $[45/0/-45/90]_s$ is used for the plate and their properties are listed in Table 1. Four different finite element meshes were used to study the sensitivity of these codes to mesh size. Figure 2 shows four different finite element meshes with 168, 400, 600 and 864 elements, respectively.

Thermal analysis is applied in the simulation using EFM-MCT code in order to take into account the ply thermal residual stress that occurs during the manufacturing/curing process of the composite. Simulations using HELIUS:MCT™ on the other hand do not include thermal analysis because HELIUS:MCT™ does not facilitate this. Attempts have been made to use ABAQUS native thermal expansion model combined in addition to Helius-MCT material model in order to model the curing process, but a converged solution cannot be obtained. Thus, another simulation using EFM-MCT without the thermal analysis is performed so that the Helius-MCT can be compared to EFM-MCT with the exactly same parameters.

The simulation results in the average stress strain curves are shown in Figure 3. They suggest that both EFM-MCT and HELIUS:MCT™ codes are not sensitive to mesh size when the mesh is reasonably fine, i.e. more than 400 elements in this case. Moreover, comparison between predicted ultimate strength with experimental result [20], as shown in Figure 4, shows that both models are able to predict the experiment tensile strength quite well with error less than 15%. Simulations using EFM-MCT that do not include thermal analysis predict a slightly higher strength than the results predicted by simulations that include thermal analysis; their difference is around 1.3% – 3.3%.

Table 1 Material properties of IM7/5250-4 composite system [20]

Thickness (mm)	0.125
Modulus in fiber direction E_1 (GPa)	172.4
Transverse moduli E_2= E_3 (GPa)	10.3
Shear moduli G_{12}=G_{13} (GPa)	5.52
Shear modulus G_{23} (GPa)	3.45
Poisson's ratios v_{12}= v_{13}	0.32
Poisson's ratio v_{23}	0.4
Longitudinal tensile strength X (MPa)	2826.5
Longitudinal compressive strength X' (MPa)	1620.0
Transverse tensile strength Y (MPa)	65.5
Transverse compressive strength Y' (MPa)	248.0
Shear strength S_{12}=S_{13} (MPa)	122.0
Shear strength S_{23} (MPa)	122.0
Coefficient of thermal expansion in fiber direction α_1(/^{o}F)	-0.2×10^{-6}
Coefficients of thermal expansion in transverse directions α_2= α_3 (/^{o}F)	13.7×10^{-6}
Temperature difference (^{o}F)	-375



(a). Mesh 1 (168 elements) (b). Mesh 2 (400 elements) (c). Mesh 3 (600 elements) (d). Mesh 4 (864 elements)

Figure 2 Finite element mesh for open-hole tension model

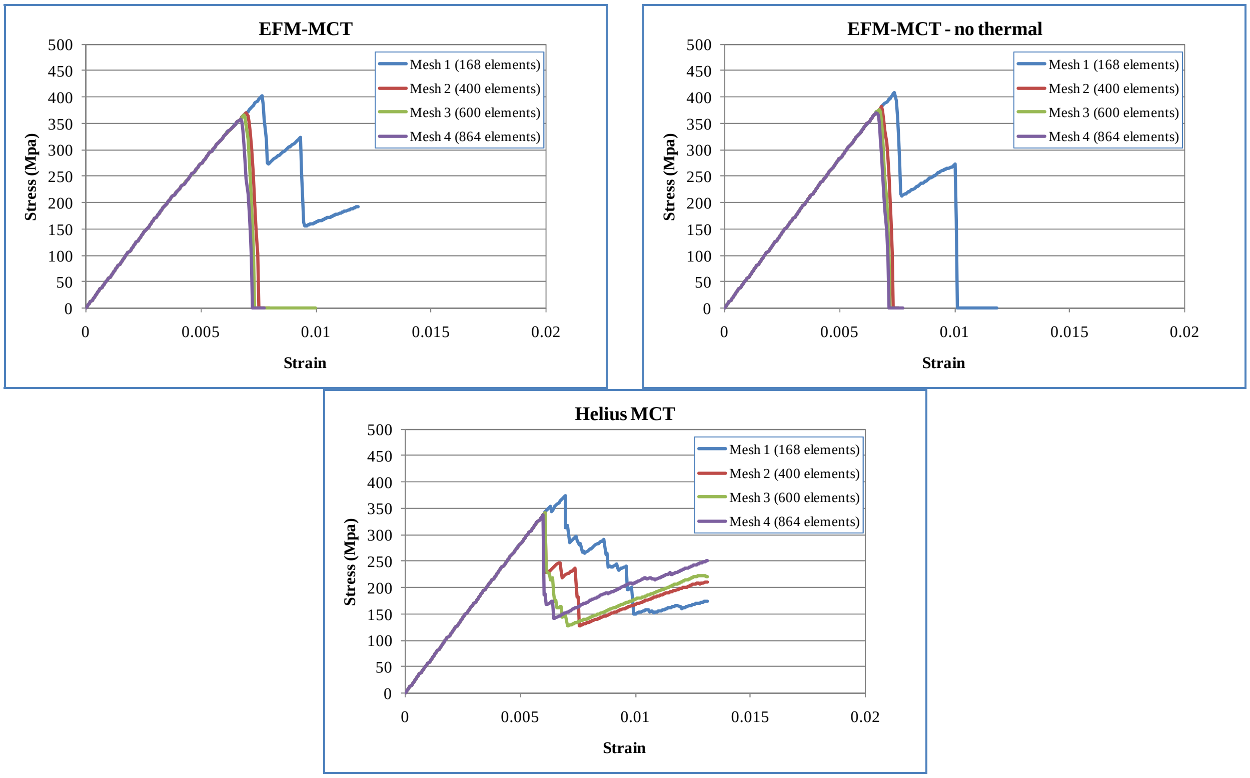


Figure 3 Stress-strain curves

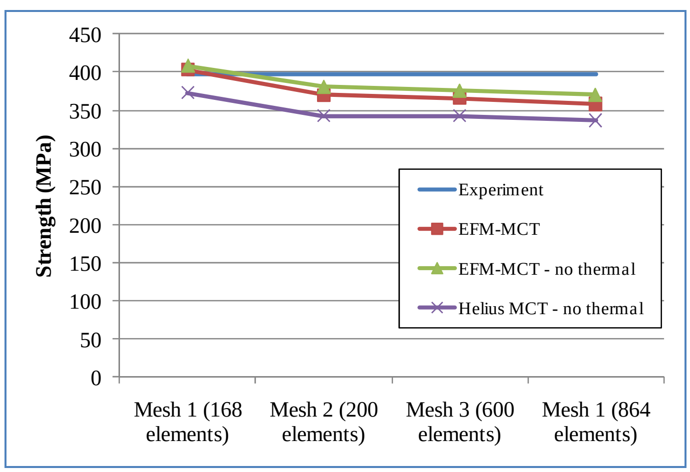


Figure 4 Strength comparison

Figures 4, 5 and 6 show the failure pattern when major load drops occur in the simulations using EFM-MCT, EFM-MCT – no thermal, and HELIUS:MCT™, respectively. Figures 4 and 5 suggest that although strengths predicted by simulations with and without thermal analysis are almost the same, their failure patterns are largely different. Simulations using thermal analysis predict a much larger matrix failed area than simulations without thermal analysis. This shows that thermal residual stress accounted for in the simulations using thermal has a major influence on matrix failure.

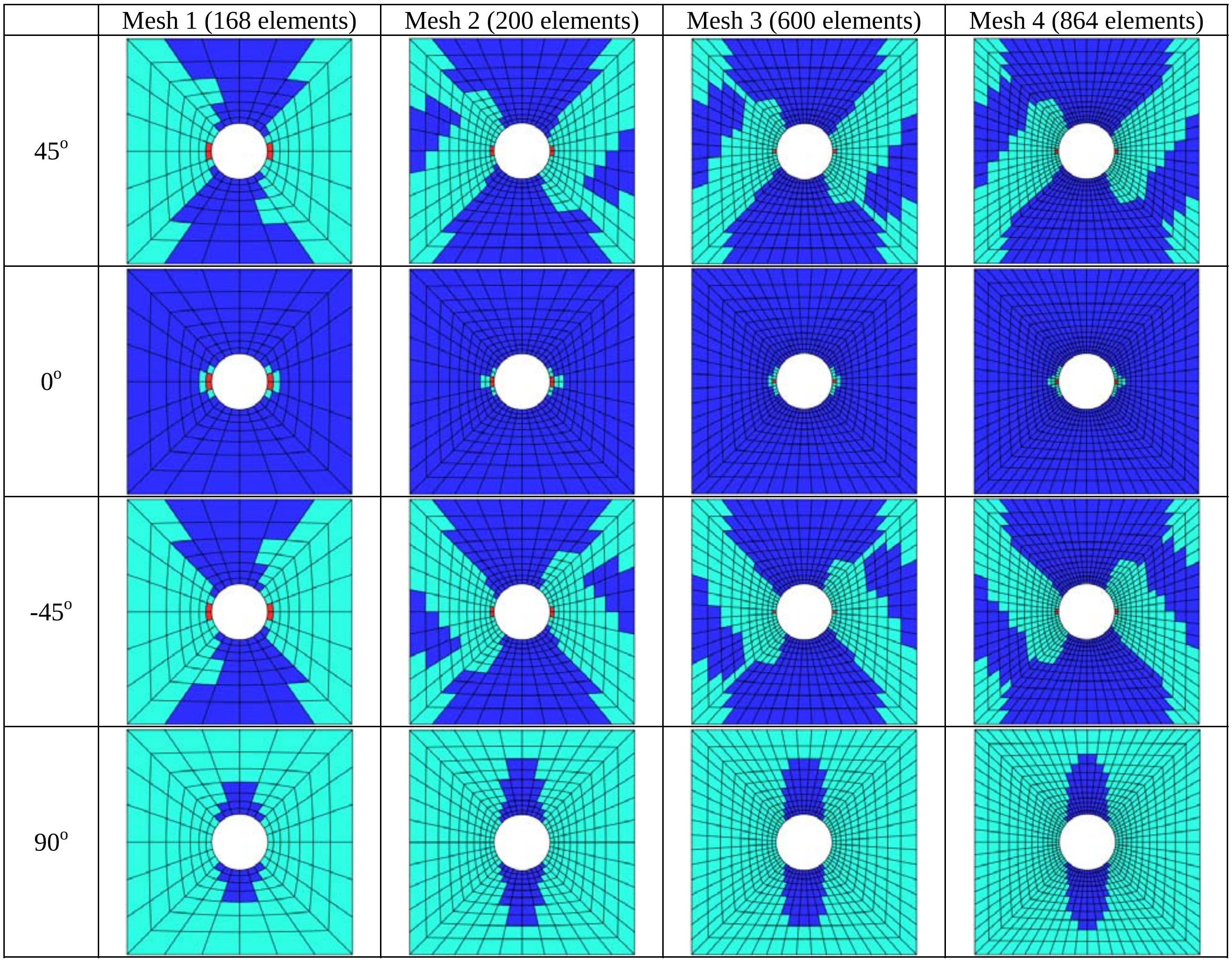


Figure 5 OHT failure pattern using EFM-MCT with thermal/curing analysis

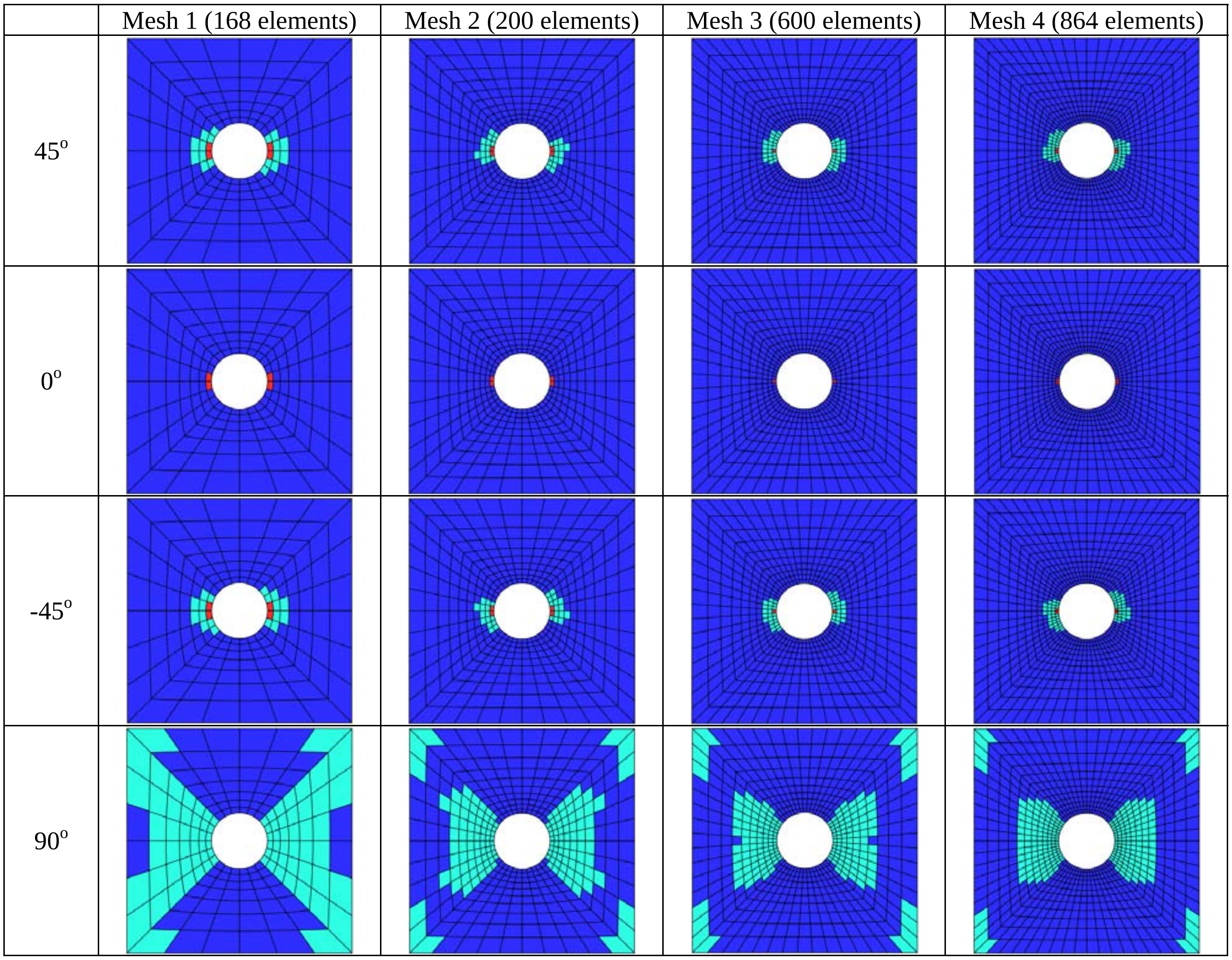


Figure 6 OHT failure pattern using EFM-MCT without thermal/curing analysis

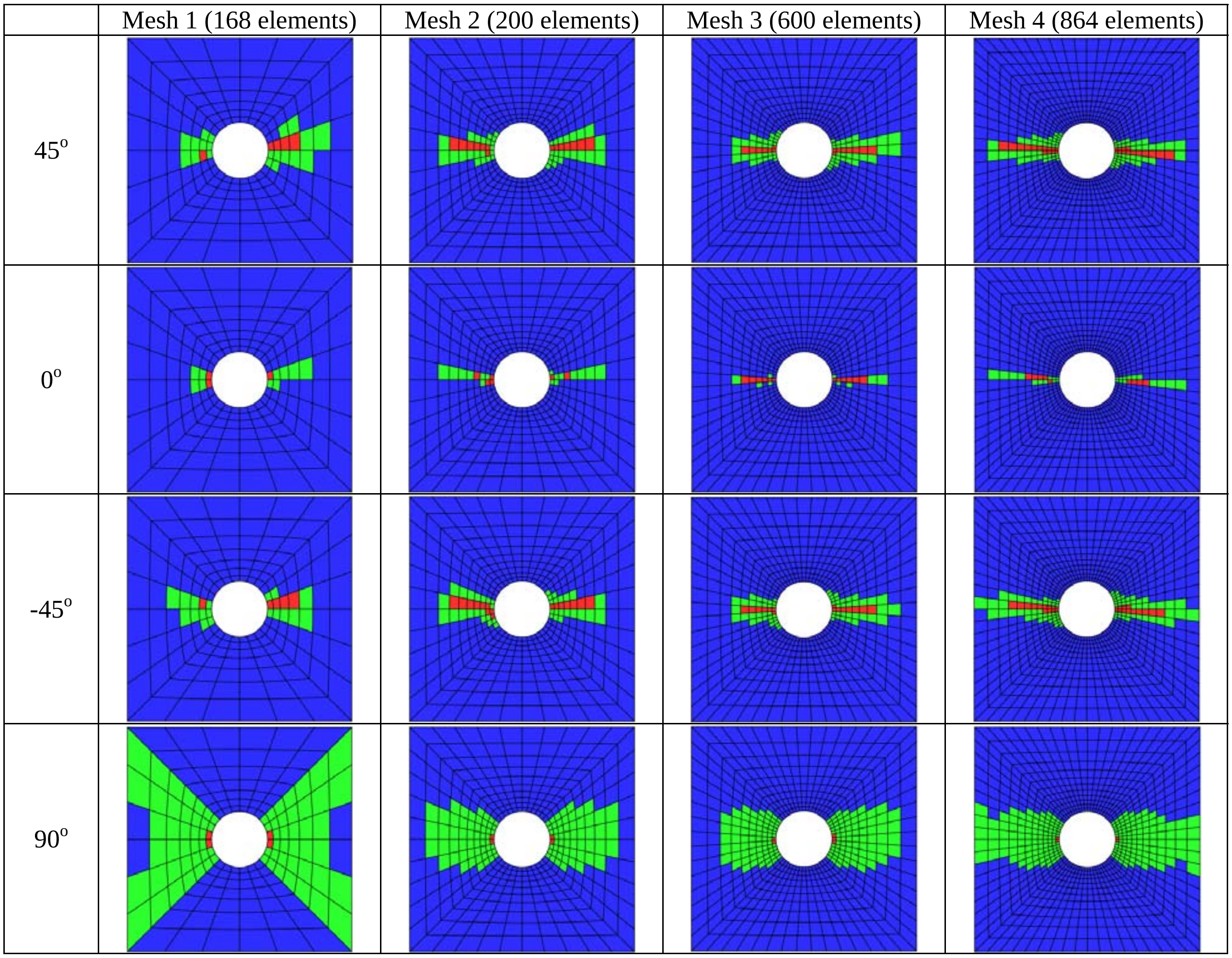


Figure 7 OHT failure pattern using HELIUS:MCT™ without thermal/curing analysis

Analysis using Mesh 4 with different number of time step was performed to compare the computation time needed by HELIUS:MCT™ and EFM MCT. Table 2 shows the comparison of CPU time needed by the two codes. Note that the simulation time for EFM-MCT consists of two parts: the first one is from the actual analysis while the second is from the dummy simulation used for visualization purpose. Table 2 suggests that HELIUS:MCT™ is more efficient in terms of CPU time compared to EFM-MCT.

Table 2 CPU time for 864 element model

	HELIUS:MCT™		EFM-MCT	
	101 time step	401 time step	101 time step	401 time step
1^{st} run (actual simulation)	691.72	1293.4	3987.4	6910.8
2^{nd} run (post processing for visualization)	N/A	N/A	73.2	180.67
total	691.72	1293.4	4060.6	7091.47

6 Comparison between N-EFM and MPDM

An example of crack propagation is used to demonstrate the computational efficiency of N-EFM, which is also compared with that of MPDM. The model and mesh are shown in Figure 8, and total 3690 3D linear brick elements with 20500 degree of freedoms (DOFs) are used in the analysis.

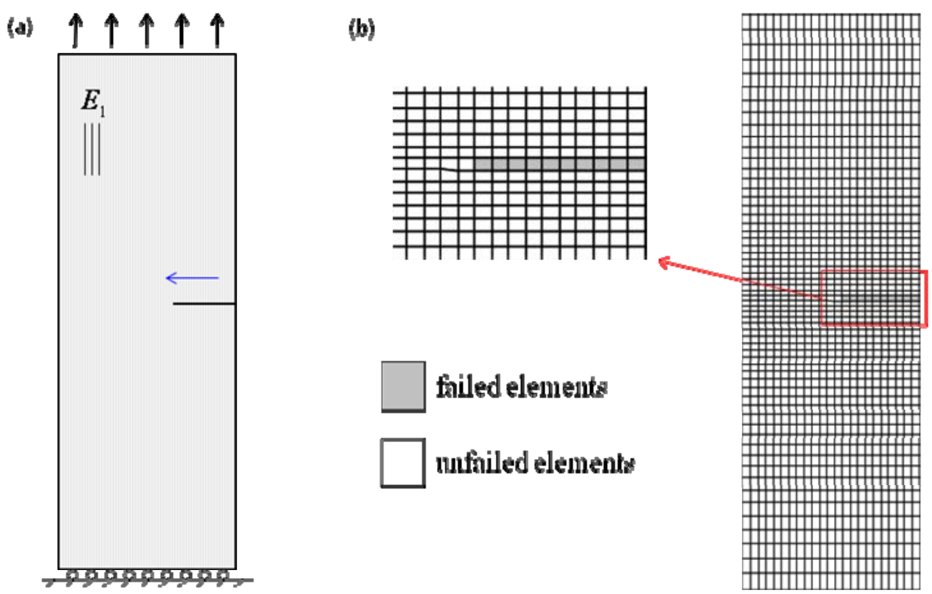


Figure 8 Example of crack propagation: (a) model; (b) mesh

Figure 9 shows the comparison of compuational efficiency of N-EFM and MPDM, with normalized compuation time by that of MPDM, where N-EFM_1 and N-EFM_2 denote results from reassembling and decomposing global stiffness matrix every 5 and 10 load increments, respectively, and 3 elements fail in each load increment. It can be seen that the efficiency of N-EFM is improved up to about 70% compared to that of MPDM.

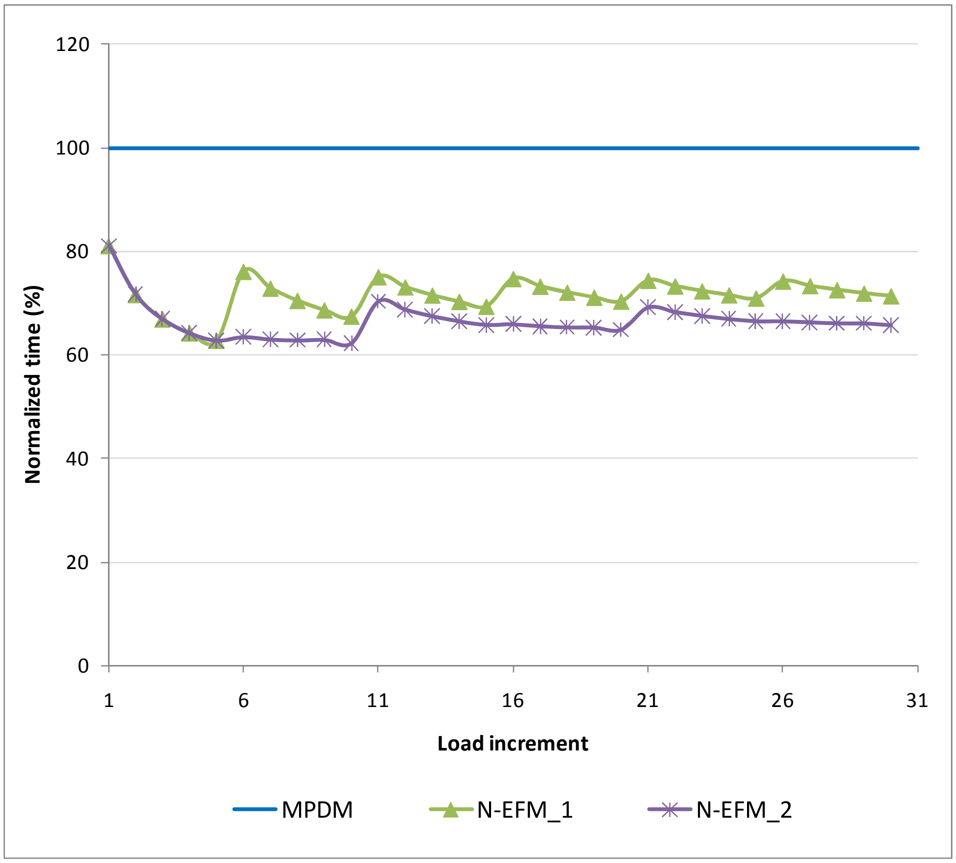


Figure 9 Comparison of compuational efficiency of N-EFM and MPDM

7 Conclusion

MCT failure criterion has been implemented in ABAQUS using the iterative element-failure method (EFM) algorithm. OHT analysis is performed using the newly developed EFM-MCT and HELIUS:MCT^{TM} which uses material property degradation (MPDM) algorithm. Results show that EFM-MCT and HELIUS:MCT^{TM} give similar results and they are in good agreement with experimental result. The EFM-MCT is however less efficient than the MPDM HELIUS:MCT^{TM}. The non-iterative element-failure method (N-EFM), which is a modified version of conventional iterative EFM, shows better computational efficiency compared to standard MPDM when the number of failed elements are small in large finite element models. When the stiffness matrix is reassembled after several load increments, N-EFM can even speed up more computational efficiency (improved up to 70%). Demonstrated concept so far is available for crack problems. We have coded MCT in our N-EFM progressive failure code. More work on demonstration of improvements in efficiency with OHT problems is on-going.

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